

## **Measurement of Statistical Errors, Iteration Algorithms and Convergence Speed in Updating Coefficient and Transaction Matrices**

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### **Abstract**

*During the past six decades, the analysts of the input – output economics (IOE) have used two approaches of input – output coefficient matrix (IOCM) and intermediate transactions matrix (ITM), both of which are based on iteration algorithms, in updating input – output tables (IOTs). The former is theoretically based on the production function and is more popular as compared with the latter which is only in terms of accounting. The challenging issue for the analysts of IOE is similar or different results of the two approaches. A group insists on different results, whereas, the observations of another group suggest that the results are equivalent. Neither of the mentioned groups consider factors such as aggregation and convergence speed with respect to the number of iterations in algorithms of the two approaches. The main focus of this article is to investigate the theoretical and empirical aspects of factors, using two survey-based symmetric IOTs of Iran for the years 1996 and 2001. With respect to the above factors, the 3 sectors, 7 sectors, 15 sectors and 21 sectors are considered. This article concludes two overall findings: a 1-statistical error*

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*in the two approaches exits but insignificant. 2-The convergence speed with lower number of iterations in the IOCM approach is higher than that of the second approach.*

**Keywords:** RAS method, input – output coefficient matrix, intermediate transactions matrix, statistical errors

**JEL classification:** C67, D57, C80

## **1. Introduction**

Experiences of compilation of IOTs in various countries proved that compilation of annual survey-based IOTs is expensive and not economical because they need detailed data such as different censuses both at national and pectoral levels, data processing and expenses for data preparation. Therefore, non-survey and semi-survey methods such as conventional RAS and adjusted RAS methods should be used to estimate such tables. As compared with the survey-based tables, such methods enjoy the advantages of least requirements of data, lowest financial and labor costs and a shorter time required to estimate symmetric IOTs.

In all non-survey methods, researchers try to convert IOTs on the basis input – output coefficients in such a way that technological and structural changes are taken into account (Mirshojaeyan and Rahbar, 2011). Generally, these methods may be placed in two main categories. The first category involves optimization methods. In these methods, updating is converted to the problem of minimization of functions of conditions which measure differences of certain elements between the matrix of the base year and that of the target year.

Therefore, in these methods, the main concern is to find some answer to this problem which takes the estimated matrix close to the target matrix (Lahr and de-Mesnard, 2004). The said conditions constitute the equality of sums of row and column of updated matrix with those of target year. The second category is Bi-proportional Adjustment Methods. These methods are based on the famous method of iteration algorithms. Therefore, they need initial estimation of the target matrix. Such estimation is usually supposed to be the initial matrix. At the second stage, this matrix is multiplied on the left side by the row adjustment matrix. At the third stage, the matrix of the second stage is multiplied on the right side by column adjustment matrix. Next the resulted matrix is placed instead of the matrix of the zero stage. The 1<sup>st</sup> and 2<sup>nd</sup> stages will be repeated as long as considered conditions are realized. Jackson and Murry (2004) indicate 10 Bi-proportional Adjustment Methods and conclude that among the Bi-proportional Adjustment Methods it is the RAS method which enjoys considered conditions and whose answers are mostly logical. Therefore, researchers widely use this method. de-Mesnard (1994), too, proved that all Bi-proportional Adjustment Methods will reach the same answer as that of RAS Method after passing iteration stages. However, RAS Method is preferred over other methods because it is simple and required less data (Lahr and de-Mesnard, 2004).

Despite popularity of RAS Updating Method, there is still exist some controversial issues about this method among the IO analysts. One challenging issue is using the approach of IOCM and/or the approach of ITM in updating IOTs. The first one considers the theoretical bases of

production function, while the second one deals only with accounting aspects. The controversial point among input – output economists is whether results of using these approaches are the same or different. A group of them observes that such results are different [1], while Dietzenbacher and Miller, (2009), and Miller and Blair( 2009), mathematically proved that it makes no difference either to use the first approach or the second approach. Neither of the two above group looked into account the issues like aggregation, the number of iteration and convergence speed of the two said approaches.

The foregoing observations bring up two important questions. The first question is: Do the results of using both approaches of TOCM and ITM are the same? Second question: Considering the different scenarios of aggregation which approach has higher convergence speed with lower iterations in updating IOTs?

In view of the above and evaluation of the research environment of updating IOTs in Iran, we reach to the two general points, the study of which may begin a new chapter in input – output research in Iran.

First point: The input – output coefficient matrix was always used as the basis of updating such tables in Iran [2].

Second point: Despite half a century experience in compiling IOTs in Iran, there are very few articles in connection with dealing with theoretical aspects of updating methods of IOTS.

To answer the above questions, we use two symmetric survey-based industry by industry tables based on the industry technology assumption for the years 1996 and 2001 which are aggregated in 3, 7, 15 and 21 sectors. We use conventional RAS Method as the basis for updating IOTs using two approaches of IOCM and ITM. The present article is organized in five parts. The first part deals with available literature in connection with the discussed issue. The second part is allocated to theoretical aspects of the conventional RAS Method in two approaches followed by three measurement methods of statistical errors of Root Mean Square Error (RMSE), Mean Absolute Deviation (MAD) and Standard Total Percentage Error (STPE). We discussed in third and fourth parts respectively data bases, empirical results and analysis. The fifth part is allocated to summary and conclusion.

## **2. Brief Review of Literature**

The years after World War II, especially the beginning years of 1950s are known to be the time of foundation and globalization of input – output economics and its applications in various fields of national and regional economies (Banouei, 1997). Then, it was felt more necessary to compile tables in national and regional levels first in developed countries and then in some developing ones (Stone, 1980). Such tables were survey-based in nature [4]. However, they have at least two major drawbacks: i.e. expensive

and also time consuming in the process of compilation. To overcome the problems of compilation of IOTs, in the beginning of 1960s Richard Stone et.al; recommended the RAS Method to update IOTs (Stone, 1961, Stone and Brown, 1962). From the View point of methodology and also data requirements, since the beginning years of 1960s the updating methods may be divided in the following three main groups:

First group: It is conventional RAS Method. The main objective of using the conventional RAS Method is to use input – output coefficient matrix of the base year as well as required data of the target year to estimate the input – output coefficient matrix of the target year. The second group includes Adjusted RAS Method. The distinguished difference between the Adjusted RAS Method and the conventional RAS Method is gathering exogenous data and/or additional data in the target year, usually belonging to the intermediate transactions and/or input – output coefficients of the target year. Available literature suggests that both the RAS Method and the Adjusted RAS Method are more popular than other methods among international institutes, statistical institutes of countries and also among researchers, because they are simple and need least data. In spite of such popularity, there is still a controversial point with RAS Method, which attracted the attention of some researchers of input – output economics: It is the similarity of results or, as some believe, differences between the approach of intermediate transaction matrix and input – output coefficient matrix in updating IOTs.

In this respect, theoretical and practical experiences of some group show that the results are the same. For example, Miller and Blaire (2009) indicated in their book that the results achieved from both approaches are similar and only in coefficient matrix approach we need to convert coefficient matrix to transaction matrix and the results are achieved directly. In the book they continued to prove their claim with a numerical example. Furthermore, Dietzenbacher and Miller (2009), too, suggest in details in their article that in terms of economic theories, if the approach of direct coefficient matrix is used as the basis of updating, the process of updating will be based on theoretical bases, because such coefficients originate from production functions, and if the approach of intermediate transactions matrix is used as the basis of updating, accounting systems will apply. They go even further to prove that the updating of direct coefficient matrix of Ghosh supply side model, like coefficient and transaction approaches give equivalent results. Finally they reach to final observations that from mathematical point of view the results of all three groups are equivalent. Okuyarma et al. (2002) indicate that the process of adjustment with the aid of coefficient matrix reaches a conservative estimation, and that such process needs technical adjustments. Consequently, they imply that use of direct coefficient matrix of demand side of Leontief is more proper than intermediate transaction matrix.

Moreover, Jackson and Murry, 2004, rely on Okuyama's studies to suggest that applications of coefficient matrix and intermediate transaction matrix are different and use of coefficient matrix will lead researchers to more acceptable results (Lahr and de-Mesnard, 2004).

The third group is Generalized RAS Methods recommended with the objective of removing some disadvantages of RAS Method during recent years. One disadvantage of RAS Method and even Adjusted RAS Method is that they are sensitive only to positive and zero cells during updating and they ignore negative cells such as net export and/or net tax in IOTs. The aim of Generalized RAS Methods, such as Scaling RAS (KRAS), Generalized RAS (GRAS), Three Stage RAS (TRAS), Cell Corrected RAS (CRAS) and Improved Generalized RAS (IGRAS) are in fact to remove such disadvantages [5].

The updating of IOTs has a long background in Iran, however, theoretical aspects, advantages and disadvantages of such methods were not considered by researchers for unknown reasons. The RAS Method and/or Adjusted RAS Method have been used for updating input – output tables by various institutes such as the then Ministry of Economy, the then Plan and Budget Organization, Ministry of Power, Central Bank of Iran, Statistical Center, and, recently, Research Center of Parliament. For instance, in 1972 Eckestein and Badakhshan, who acted upon an order by the then Ministry of Economy, and with the aim of quantitative study of import substitutions in the 5<sup>th</sup> five-year plan before the Islamic Revolution, succeeded to update IOTs of the years 1971-1977 of Iran on the basis of IOT of the year 1965 using the Conventional RAS Method (Eckestein and Badakhshan, 1972). The then Ministry of Plan and Budget updates the IOT of the year 1984 on the basis of the IOT of the year 1974 (Ministry of Plan and Budget, 1989). Probing into methodological aspects of various non-survey IOTs in Iran, we see that the approach of input – output coefficient matrix has always been used as the basis of calculation with no attention to the approach of intermediate transaction matrix and also without taking into account factors such as aggregation and convergence speed of the two approaches in updating IOTs. As far as our knowledge permits, these issues are considered a lacuna for input-output research.

### **3. Theoretical bases**

As it was stated in the introduction, RAS Method is very popular among researchers of IOTs. However, it should also be noticed that this method can lead to precise estimations of IOTs only when there are precise estimations of outputs of sectors, initial inputs and final demands, because, when there is any error in data, imprecise estimations may be achieved.

Therefore, if we suppose that there are precise data, the only problem any researcher should pay attention is to rely on the direct coefficient matrix and/or intermediate transaction matrix of the base year.

### 3.1. Theoretical bases of RAS Method

RAS method deals with three non-negative matrices, say,  $A(0) = (a_{ij}^0)$ ,  $A(1) = (a_{ij}^1)$  and  $\tilde{A}(1) = (\tilde{a}_{ij})$  of the same size  $m \times n$ . The matrix  $A(0)$  is given and is called intermediate transaction matrix or table of the base year  $t = 0$ . The matrix  $A(1)$  is called transaction matrix of target year  $t = 1$  and its row and column totals are given as well:

$$\sum_{j=1}^n a_{ij}(1) = u_i(1) \quad i = 1, 2, \dots, m \quad (1)$$

$$\sum_{i=1}^m a_{ij}(1) = v_j(1) \quad j = 1, 2, \dots, n \quad (2)$$

No more information about  $A(1)$  may be given.  $\tilde{A}(1)$  is an estimation of  $A(1)$  which should be determined using  $A(0)$  in such a way that its row and column totals are row and column totals of  $A(1)$  respectively, i.e.

$$\sum_{j=1}^n \tilde{a}_{ij}(1) = u_i(1) \quad i = 1, 2, \dots, m$$

$$\sum_{i=1}^m \tilde{a}_{ij}(1) = v_j(1) \quad j = 1, 2, \dots, n$$

And is close to  $A(1)$  in a sense to be described later.  $\tilde{A}(1)$  is also called updated matrix of  $A(0)$ . Note that during this process,  $A(1)$  does not change and it may remain unknown forever.

Stone (Bacharach, 1970) presented this problem in the field of input – output economy and solved it using an iteration method. Suppose that  $x = (x_1, \dots, x_n)$  is a vector with  $n$  entries. The symbol  $\langle x \rangle$  will designate a diagonal matrix with  $x_1, \dots, x_n$  as its main diagonal entries. The solution for the problem was the bi-proportional matrix:

$$\tilde{A}(1) = R_A A(0) S_A$$

In which  $R_A = \langle r \rangle$  and  $S_A = \langle s \rangle$  are appropriate diagonal matrices, and  $\tilde{A}(1)$  satisfies all conditions of the problem.

Instead of updating the intermediate transactions matrix, one may update direct coefficients of Leontief's demand side model. This matrix for  $t = 0$  is denoted by  $L(0) = (l_{ij}(0))$ . The relationship between the initial matrices  $L(0)$  and  $A(0)$  are specified using the vectors of final demand  $f(0)$  and gross supply  $x(0)$ . In fact we have:

$$l_{ij}(0) = \frac{a_{ij}(0)}{x_j(0)} \quad i = 1, \dots, m \quad j = 1, \dots, n$$

Or in matrix notation

$$L(0) = A(0) \langle x(0) \rangle^{-1}$$

Corresponding to the matrix  $L(0)$  and the vector  $x(0)$  are the matrix  $L(1) = (l_{ij}(1))$  and vector  $x(1) = (x_1(1), \dots, x_n(1))$  for  $t = 1$ .

As above we have:

$$\begin{aligned} L(1)x(1) &= (u_1(1), \dots, u_m(1)) \\ \acute{e} l(1)\langle x(1) \rangle &= (v_1(1), \dots, v_n(1)) \end{aligned}$$

Updating  $L(0)$ , using RAS method and restrictions above we obtain the matrix  $\tilde{L}(1)$  that satisfies the relations:

$$\begin{aligned} \tilde{L}(1) &= R_L L(0) S_L \\ \tilde{L}(1)x(1) &= (u_1(1), \dots, u_m(1)) \\ e^{\tilde{L}(1)} &= (v_1(1), \dots, v_n(1)) \langle x(1) \rangle^{-1} \end{aligned} \quad (4)$$

Now we transform  $\tilde{L}(1)$  into a transaction matrix, i.e. define:

$$\tilde{A}_t(1) = \tilde{L}(1) \langle x(1) \rangle$$

And conversely transform  $\tilde{A}(1)$  into a coefficient matrix, i.e. define:

$$\tilde{L}_t(1) = \tilde{A}(1) \langle x(1) \rangle^{-1}$$

A theorem from Diazenbacher and Miller (2009), states that:

$$\tilde{A}_t(1) = \tilde{A}(1) \text{ and } \tilde{L}_t(1) = \tilde{L}(1)$$

### 3.2. Methods of measurement of errors

To determine efficiencies and evaluate performances of the mentioned approaches, it is required to compare tables resulting from the two updating approaches with the statistical input – output tables of the target year. The closer these tables to each other, the better performance of the approach.

Among numerous methods of calculation of statistical errors, there is no indication that one of the methods is preferable comparing to the others. Therefore, we use three methods of estimation of errors, namely, Mean Absolute Deviation (MAD), Root Mean Square Error (RMSE), and Standard Total Percent Error (STPE), for which general expositions exist. Using the above notation we briefly describe each of these methods.

The method of Mean Absolute Deviation (MAD) measures the mean total absolute errors of entries of updated matrix from the entries of the target matrix. Therefore, it describes total statistical errors and in each case is defined as follows:

$$MAD^A = \left( \frac{1}{m \times n} \right) \sum_i \sum_j |a_{ij}(1) - \tilde{a}_{ij}| \quad (5, a)$$

$$MAD^L = \left( \frac{1}{m \times n} \right) \sum_i \sum_j |l_{ij}(1) - \tilde{l}_{ij}| \quad (5, b)$$

The method of Root Mean Square Error (RMSE), as compared with the method of Mean Absolute Deviation (MAD), does not yield any idea of the relative difference between two matrices, but rather only the average total difference.



$$\text{RMSE}^A = \sqrt{\frac{\sum_i \sum_j (a_{ij}(1) - \tilde{a}_{ij})^2}{m \times n}} \quad (6, a)$$

$$\text{RMSE}^L = \sqrt{\frac{\sum_i \sum_j (l_{ij}(1) - \tilde{l}_{ij})^2}{m \times n}} \quad (6, b)$$

The method of Standard Total Percent Error (STPE) goes back to Leontief in input-output economy and is defined as follows:

$$\text{STPE}^A = \frac{\sum_i \sum_j |a_{ij}(1) - \tilde{a}_{ij}|}{\sum_i \sum_j a_{ij}(1)} \quad (7, a)$$

$$\text{STPE}^L = \frac{\sum_i \sum_j |l_{ij}(1) - \tilde{l}_{ij}|}{\sum_i \sum_j l_{ij}(1)} \quad (7, b)$$

#### 4. The Data base

As it was mentioned before, one of the basic principles of using updating methods is to take into account the compatibility and consistency of statistical basis, no matter which approach is used. For this purpose, we have used two survey-based IOTs of the years 1996 and 2001. The 1996 table is derived on the basis of make and use tables from Social Accounting Matrix (SAM) of the year 1996 (National Research Project, 2002), a symmetric industry-by-industry using industry technology assumption [6]. For compatibility and consistency of the table of the year 2001 with the table of the year 1996, the make and use table of the year 2001 of the Statistical Center of Iran, a symmetric industry-by-industry with industry technology assumption has been derived. In line with objectives and questions of the article, the above tables were aggregated in 3, 7, 15 and 21 sector tables as follows [7]. The 21-sector table includes sectors of "agriculture, husbandry and forestry", "animal husbandry, aviculture, breeding silk worms and honey bees, hunting and fishing", "petroleum and natural gas", "other mines", "food and beverage industries, tobacco, textile industries, clothing and leather", "wood, paper and printing industries", "chemical, rubber and plastic industries", "non-metal mineral industries", "other industries", "provision of electricity, water and gas", "construction", "wholesale and retail sale and maintenance of vehicles and home appliances", "hotel and restaurant", "transportation, storage and communications", "financial intermediaries", "real estates, leasing, and business services", "public affairs management, defense and social service", "education", "public health and social relief" and "other activities". The 15- sector table includes sectors of "animal husbandry, aviculture, breeding silk worms and honey bees, hunting and fishing", "petroleum and natural gas", "other mines", "food and beverage industries, tobacco, textile industries, clothing and leather", "other industries", "provision of electricity, water and gas", "construction",

"wholesale and retail sale and maintenance of vehicles and home appliances", "hotel and restaurant", "transportation, storage and communications", "financial intermediaries", "real estates, leasing, and business services", "public affairs management, defense and social security", and "other activities". The 7- sector table includes sectors of agriculture, oil, mines, provision of water, electricity and gas, construction and services. Finally the 3- sector table includes three main sectors of agriculture, industry and services. At the following stage, with the aid of Excel software, the survey-based symmetric tables of the year 1996, as the base year, data on production vectors, intermediate demand and intermediate cost in the symmetric table of the year 2001 have taken as target year, to analyze empirically two approaches. Then, in view of the objectives and questions of the article and using three conventional methods of measurement of statistical errors like Mean Absolute Deviation (MAD), Root Mean Square Error (RMSE), and Standard Total Percent Error (STPE), the results of the two approaches of the input – output coefficient matrix and the intermediate transaction matrix updated in the year 2001 with corresponding actual data of the same year were evaluated in terms of 3, 7, 15 and 21 sectors.

### **5. Empirical Results**

Results from the measurement of statistical errors between the updated matrices (the intermediate transaction matrix and the input – output direct coefficient matrix) for the year 2001 in terms of 3, 7, 15 and 21 sectors with corresponding actual of the same year as well as the convergence speed of the input – output coefficient matrix and the intermediate transaction matrix are organized respectively in tables 1 and 2.

Results of the extracted statistical errors are organized in table 1. Figures of statistical errors in the method of Mean Absolute Deviation (MAD) show that the last decimal figure in the 3-sector and 7-sector tables and two last decimals figures in the 15-sector and 21-sector tables in the approach of the input – output direct coefficient matrix are less than the corresponding figures in the approach of the intermediate transaction matrix. Figures resulted in the method of Root Mean Square Error (RMSE) show also a similar view. For example, statistical errors of the last decimal figure and two last decimal figures of the 3-sector and 15-sector tables in the approach of coefficient matrix are less than the corresponding figures in the approach of the intermediate transaction matrix. Nevertheless, statistical errors of the last four decimal figures in the 7-sector and the three last decimal figures of in the 21-sector tables are more than corresponding figures in the approach of the direct coefficient matrix. The differences of errors in the Standard Total Percentage Error (STPE) are more significant. In this case, results of the mentioned method show that the statistical errors of the last four decimal

figures in the 3-sector and 7-sector tables and the last six figures and seven figures respectively in the 15 and 21 sector tables extracted by the approach of the input – output direct coefficient matrix are less than corresponding figures of the approach of the intermediate transaction matrix.

The above results and observations reveal at least two facts about the existing challenge in connection with the equivalent results of the two approaches: First, although the statistical errors between the two approaches are negligible, however, the outcomes of the two approaches do not prove that they are equivalent. In this case, statistical errors shows that in all aggregated tables of the approach of input – output direct coefficient matrix are less than those of the approach of intermediate transaction matrix. Second, such differences in the approach of input – output direct coefficient matrix are not only lower than those of the approach of intermediate transaction matrix, but also errors will grow higher in number in the three methods of measurement of errors as the number of sectors gets higher.

Another important issue is the neglect of role and importance of the number of iterations and, consequently, the convergence speed of two approaches among the input – output analysts. The results of convergence speed of matrices in Table 2, show that the degree of convergence speed of aggregated sectors are higher than the corresponding Transaction approach. For example, figures in Table 2, Column 1, indicate that the number of iterations in converting input-output coefficient for 3, 7, 15 and 21 sectors are respectively, 7, 9, 11 and 21 whereas corresponding figures for Transaction matrix are, 8, 10 and 23. These findings illustrate at least three facts for the compilers as well as users of IOTs (1) The convergence speed with lower number of iteration in the first approach is higher than that in the second one with higher number of iteration and, in terms of time saving, the first approach is preferred to be used for the updating of input – output tables (2) The issue of aggregation may not be overlooked. For instance, the difference between the numbers of iteration in the two approaches is one in both 3 and 7 sector tables. However, such difference amounts to 5 for 15-sector tables and to 2 for 21-sector tables. As we explained under note 7, because of data limitations, it is impossible to consider more numbers of sectors (than 21 sectors). Therefore, it is impossible to conclude definitely whether aggregations reduce differences in the convergence speed between the two approaches, or not. (3) Another important point to be considered by analyzers of the input – output researchers is that more numbers of iterations which has taken here as convergence speed, may lead to the divergence of matrix in the approach of the intermediate transaction matrix. To solve this problem, analysts are forced to aggregate some of the sectors. However, it is expected that the use of the input – output coefficient matrix does not have such disadvantages [8].

Table 1- Statistical errors resulted from two approaches

	Input-output coefficient matrix approach					Intermediate transaction matrix approach						
	3 SECTOR	7 SECTOR	15 SECTOR	21 SECTOR	3 SECTOR	7 SECTOR	15 SECTOR	21 SECTOR	3 SECTOR	7 SECTOR	15 SECTOR	21 SECTOR
<b>MAD</b>	0.004832611	0.009288016	0.006911642	0.008954957	0.004832611	0.009288017	0.006911649	0.008954957	0.004832611	0.009288017	0.006911649	0.008954957
<b>RMSE</b>	0.011203568	0.044705699	0.047915558	0.101173618	0.011203568	0.0447057	0.047915718	0.101173626	0.011203568	0.0447057	0.047915718	0.101173626
<b>STPE</b>	4.075652278	17.59786863	29.91556481	56.15317532	4.075652368	17.59786885	29.91559378	56.15317772	4.075652368	17.59786885	29.91559378	56.15317772

Source: Based on the symmetric tables of 3,7,15 and 21 sectors of initial year 1996 and the target year 2001, using two approaches of coefficient and transaction matrices.

Table 2- Convergence speed of matrices in two approaches

	Number of iterations and convergence speed of input-output coefficient Matrix: approach(1)	Number of iteration and convergence speed of intermediate transaction Matrix: approach(2)
<b>3 SECTOR</b>	7	8
<b>7 SECTOR</b>	9	10
<b>15 SECTOR</b>	11	16
<b>21 SECTOR</b>	21	23

Source: Based on the symmetric tables of 3,7,15 and 21 sectors of initial year 1996 and the target year 2001, using two approaches of coefficient and transaction matrices.

## **6. Summary and conclusion**

During the last six decades, analysts of the input – output economics have used two approaches to update input – output tables; the Approach of input – output direct coefficient matrix and the approach of intermediate transaction. Do both approaches lead to similar results or not? This has become a controversial issue among the input – output analysts. A group insists on the different results. However, another group's findings show the equivalent results. Neither of the two said groups has taken into account factors such as aggregation, the nature of iteration numbers and convergence speed of matrices in the two approaches. In this article, we intend to fill this lacuna and examine the said problem with raising two basic questions. First question: As results achieved with the application of the two approaches of the input – output direct coefficient matrix and the intermediate transaction matrix the same? Second question: In view of the aggregation of sectors, which one of the two approaches has lower iteration number with higher convergence speed in updating of IOTs? To evaluate the two raised questions, we used two survey-based Iranian IOTs as follows: The symmetric industry-by-industry tables with industry technology assumption for the years 1996 and 2001. In connection with the first question, the three methods of Root Mean Square Error (RMSE), Mean Absolute Deviation (MAD) and Standard Total Percentage Error (STPE) were used to measure statistical errors between the two approaches with 3-sector, 7-sector, 15-sector and 21-sector tables. The results show that the errors exist but are insignificant. In addition to that we observe that these errors are relatively (not clear cut) sensitive to the degrees of aggregation. To answer the second question, we used the criteria of role and importance of numbers of iterations and, consequently, the convergence speed of matrices of the two approaches. The findings show that the convergence speed of the approach of the input – output direct coefficient matrix is higher with lower number of iteration than that of the approach of the intermediate transaction matrix with the higher number of iteration and lower convergence speed. On the basis of these results, we recommend to compilers and users of IOTs to use the first approach in updating IOTs.

**Notes**

[1] For more information about theoretical aspects and findings of this group see:

Okuyama et al. (2002), Jackson and Murray (2004) and Oosterhaven (2005)

[2] For more information refer to the Research Center of the Parliament (2012), and Statistical Center of Iran (1991)

[3] The authors of this article observe that there are only two articles in this field in Iran: the article of Mirshojaeian and Rahbar (2012), and the articles of Moshfegh et al. (2013) which is in process of evaluation.

[4] Survey-based tables mean that tables are compiled on the basis of detailed surveys, censuses and registered data. Non-Survey-based tables mean that tables are updated on the basis of Survey-based tables of base years and data of target years making use of various updating methods.

[5] The present article doesn't aim to study such issues. For more information about theoretical aspects, data requirements as well as applications of such methods, see:

Junius and Oosterhaven (2003), Oosterhaven (2005), Huang et.al (2003), Lemelin (2009), Lezen et al. (2009), Temurshove and Miller (2013), Oosterhaven et al (2008)

[6] The social accounting matrix of the year 1996 was calculated in collaboration with the Central Bank of the Islamic Republic of Iran, Statistical Center of Iran and Faculty of Economics of Allameh Tabataba'i University.

[7] The Symmetric industry-by-industry table of the year 1996 based on the industry technology assumption contains only 21 sectors. Therefore, no more than 21 sectors may be considered. On the other hand the Symmetric industry-by-industry table of the year 2001 based on the industry technology assumption contains 99 sectors, which are aggregated on the basis of 3, 6, 15 and 21 sector tables of the year 1996.

[8] on account of limited number of sectors in the 1996 Symmetric Table, the theoretical and practical aspects of this issue could not be considered here and requires separated efforts.

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