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Rational Expectations, the Lucas Critique and the Optimal Control of Macroeconomic Models: A Historical Analysis of Basic Developments in the 20th Century

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In this paper, we first consider the role of rational expectations, the Lucas critique and the policy ineffectiveness debate in economic applications of optimal control theory. The problem of time-inconsistency in optimal control of macro-economic models with rational expectations will then be analyzed. The impact of reputation and the stochastic environment on the problem of inconsistency in dynamic choice together with the question of how can the developments in optimal control of macroeconomic models with forward-looking expectations contribute to the practice of econometric model building are the other topics which are discussed. We have adopted a historical approach in this paper, and the scope of our analysis is confined to the basic contributions made in the 20th Century.

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1. Introduction

Most definitions of economics share the idea that economic analysis deals with the allocation of given means for the optimum satisfaction of given ends. In this sense, an economic system can be regarded as a closed system with given means defined as a bounded control space and satisfaction represented by a performance criterion. From a mathematical point of view, the method of optimal control can, in

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principle, effectively solve this problem. More specifically, optimal growth theories as well as stabilization policies possess the characteristics which make the application of optimal control theory more demanding. Optimal growth theory is concerned with the optimal choice among alternative trajectories along which an economic system can be transformed from a given initial position to a desired state at the end of a specified (or unspecified) horizon, where each trajectory is generated by applying a set of feasible controls. The theory of stabilization policy deals with government actions in dampening unwanted fluctuations and at the same time driving an economic system along a desired path. According to modern optimal control theory, an admissible stabilizing control should possess an optimizing character. This has made the application of modern optimal control theory to economic growth and planning even more productive, for an economic stabilization program with no optimality condition may not guarantee an optimum design for an economic system.

The rational expectation hypothesis which significantly changed the way in which economic policies were perceived, provided a strong criticism of the application of control theory to economic policy optimization. Economic agents respond usually not to the signals which are mechanically generated by the controller in an engineering type environment but to their own expectations of economic state variables. Rational forward-looking expectations, in contrast to the case where expectations are functions of the past behavior, make serious difficulties in standard formulation of policy optimization. Policies which are believed to be optimal *ad hoc* will become sub-optimal upon realization. This problem suggests the utilization of dynamic game theory between the controller and the agent.

Section 2 considers the role of the rational expectations, the Lucas critique and the policy ineffectiveness debate in economic applications of optimal control theory.

Since the standard dynamic programming does not accommodate the impact of future policies on current values of state variable, the principle of optimality breaks down for optimal control of non-causal or forward-looking models in which current state variables depend on the anticipated future states. Section 3 examines the very important problem of time-inconsistency in the optimal control of macro-econometric models with rational expectations. Section 4 deals with the question of the impact of reputation and the stochastic environment on the problem of inconsistency in dynamic choice. The interesting question of how can the recent developments in optimal control of macroeconomic models with forward-looking expectations contribute to the practice of econometric model building is discussed in Section 5. And finally, Section 6 provides the summary and concluding remarks.

2. Rational Expectations, the Lucas Critique and the Policy Infectiveness Debate

In an engineering approach to economic applications of optimal control theory, it is usually assumed that the behavior of an economic agent does not depend upon the anticipation of future events including the future course of policy actions. This is not the case, however, for optimal control applications to economic systems where the private sector's expectations of government's future decisions play a significant role. Although expectations of future values of endogenous variables have long been recognized as an important topic in macro-econometrics, the traditional process in expectations formation has been usually confined to the conventional backward-looking dynamic or distributed lag models. The rational expectations hypothesis has shifted the expectations formation process from an essentially backward-looking to a forward-looking perspective.

In order to explain how expectations are formed, Muth (1961, p.315) has advanced the hypothesis that they are essentially the same as the predictions of the relevant economic theory. He maintains that "expectations of firms (or, more generally, the subjective probability distribution of outcomes) tend to be distributed, for the same information set, about the prediction of the theory (or the objective probability distribution of outcomes)".[1] Muth's paper, which has motivated a rich literature on the subject, is largely based on his earlier paper (1960) in which he established an economic argument to rationalize the "adaptive expectations mechanism" advanced by Milton Friedman (1956) and Phillip Cagan (1956). The rational expectations hypothesis was then developed further and extended by

1. Keuzenkamp (1991) refers to a paper by Tinbergen (1932), published in German, and also Tinbergen (1933) which anticipate much of Muth's analysis on rational expectations. The work of Grunberg and Modigliani (1954) should also be mentioned. They showed that economic agents can react to forecasts which might alter the course of events.

the work of Lucas (1965, published in 1981; also 1966, published in 1981) and Lucas and Prescott (1971)[1]

Estimation methods available for macroeconomic models with rational expectations are usually based on the assumption that expectations variables coincide with the future solution values over a sequence of periods. In other words, treating expectations variables explicitly rather than substituting them by appropriate distributed lag functions, necessarily requires that expectations coincide with the conditional expectations of variables based on the model itself and on all the available information (model-consistent expectations).[2]

Recent advances in econometric models with rational expectations imply that this hypothesis mainly affects short-run properties of models. Wallis (1995) reports that "as methods of estimating rational expectations models have been incorporated into large-scale macro-econometric modeling practice, it has become clear that various important long-run model properties do not depend on the choice between forward and backward-looking dynamic equations; rather, this choice principally affects the model's short-run properties" (p. 342-343). Bikker, van EIs and Hemerijck (1993) confirm this conclusion by estimating a Dutch model with rational expectations.[3]

The structure of the information set and the cost of acquiring more information are of prime theoretical concerns. It is usually assumed that individuals know the structure of the entire model as well as the historical values of all relevant variables. The rational expectations hypothesis implies that individuals do not make systematic forecast errors since the information set available to them includes the past errors. Since expectations are forecasts conditional upon the set of available information, the prediction errors are orthogonal to the information set. In a stochastic environment this means that the

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1. For comprehensive and critical work on early contributions in rational expectations hypothesis see Shiller (1978), Fischer (1980), Begg (1982), Minford and Peel (1983) and Sheffrin (1983). For early contributions of the rational expectations hypothesis on macroeconomic policy and particularly on monetary policies, see Sargent (1973, 1977), Sargent and Wallace (1973, 1975, 1976), Barro (1976), Fischer (1977) and McCallum (1977).
2. For contributions to estimation of econometric models with rational expectations see Muth (1960, published in 1981), Lucas and Sargent (1979), Blanchard and Kahn (1980), Hansen and Sargent (1980), Chow (1980), Wallis (1980) and Pesaran (1987). Pagan (1986) provides a survey of the appropriate estimation methods for macro-econometric models with rational expectations.
3. For problems associated with macroeconomic policy formulation using large econometric rational expectations models, see Christodoulakis, Gaines and Levine (1991).

unobservable subjective expectations are exactly the mathematical conditional expectations which are derived from the model known to the individual.

The implications of rational expectations for policy analysis are remarkably significant. Unlike conventional backward-looking models, models with forward expectations differentiate between anticipated and unanticipated policy changes. In the case of anticipated shocks, some responses prior to the actual shock might be expected. The same argument applies for temporary and permanent shocks. In a backward-looking environment, the model tends back to its original state when the temporary shock is removed. However, forward-looking expectations allows individuals to change their current behavior on the basis of anticipating the future removal of a temporary shock.

Lucas (1972a,b) has received the credit for applying the rational expectations hypothesis to macroeconomic models. However, the full impact of the rational expectations hypothesis on economic policy analysis and optimization did not take place until the work of Sargent (1973), Sargent and Wallace (1975), Barro (1976), Lucas (1976) and Kydland and Prescott (1977). Their work initiated the debate known as *policy ineffectiveness* in models embodying rational expectations. For example, Sargent and Wallace (1975, p. 242) demonstrated that in models with rational expectations "the probability distribution of output is independent of the deterministic money supply rule in effect". In other words, the anticipated systematic monetary policy could have no effect on the mean and variance of output. This is a very strong result. It generalizes the claim of Friedman (1968) that in the long-run output was independent of monetary policy: Sargent and Wallace (1975) concluded that output was independent of monetary policy even in the short run.

A number of research work, based on the absence of sufficient future or contingent markets, were delivered to demonstrate that the anticipated systematic monetary policy does have some effects on output (see, for example, Buiter 1981). Also Fischer (1977) provides a model with rational expectations (based on sticky wages) in which systematic monetary policy can be used to stabilize the economy. However, Holly and Hallett (1989) have shown that the neutrality proposition of Sargent and Wallace is essentially associated with the concept of *controllability* in optimal control theory. In this context, the question of existence of optimal policy can be reduced to the controllability conditions in models with rational expectations[1]. The work of Sargent and Wallace (1975) was significantly influential not only for its profound contribution to the policy ineffectiveness debate but for illustrating the case where the solutions to macro- econometric models with rational expectations could be substantially different from the solutions obtained otherwise.

The assumption of optimizing behavior in expectations formation which is inherent in the rational expectations hypothesis ensures that the systematic forecast errors associated with alternative hypotheses (such as adaptive, regressive or extrapolative expectations which relate expectations on future values to past observations) are avoided. Although one might prefer a Bayesian predictor based on explicit optimizing behavior, the comparative convenience of empirical implementation can be considered as the strength of rational expectations hypothesis. However, the rational expectations hypothesis does not theoretically address a number of important issues such as the followings: How does an economic agent construct the true structure of the economy used in forming the rational expectations? (particularly when there is not a general agreement amongst economists on how the economy functions in the real world). What are the learning and revision mechanisms by which economic agents optimize the process of expectations formation to avoid systematic forecast errors?

The work of Lucas (1976) fundamentally changed the process of policy evaluation. He demonstrated that the effects of different policy regimes on the reduced form coefficients of an econometric model, arising from private sector's expectations, were ignored in the conventional (backward-looking) approach. The economic agents' expectations play an important role in the Lucas critique. The structure of an econometric model depends on the optimal decisions of economic agents. Expected future behavior of control variables effectively influences such optimal decision rules. Since expected values of policy variables vary with changes in policy regimes the structure of an econometric model will become dependent upon the

1. Given a model of an economy and given the policy sequence at the disposal of the policy-maker, the controllability is defined as whether it is possible to reach any desired policy objectives. Controllability is the necessary condition for the existence of optimal policies. See, for example, Kwakernaak and Sivan (1972).

policy rules. It should be noted that, as Buiter (1980, pp. 35-36) has observed, the Lucas critique does not necessarily rely on rational expectations; it is essentially based on the assumption that agents form expectations on their perceptions about the policy regimes.[1]

More specifically, the Lucas critique effectively refers to a significant weakness of econometric simulations to obtain predicted values of state variables to provide guidance for policy decisions. The essence of the argument is that the true parameters in an econometric model may vary with alternative policy sequences. For example, Lucas (1976) shows how a change in the variance of the money supply (a policy parameter) will change the slope of the Phillips curve (a structural parameter), implying that parameters of macro-econometric models that appear structural may not be invariant to changes in policy. Hence, Lucas (1976, p. 20) concludes that "simulations using these models, can, in principle, provide no useful information as to the actual consequences of alternative economic policies ... [This is] based not on deviations between estimated and *true* structure prior to a policy change but on the deviations between the prior *true* structure and the *true* structure prevailing afterwards".

Herein, stands the basic Lucas critique on the theory of economic policy. The classical methods of estimating "structurally stable relationships" suitable for simulation with alternative policy sequences presupposes the invariant character of estimated structural relations to policy rules: "To assume stability of [an econometric model] under alternative policy rules is thus to assume that agents' views about the behavior of shocks to the system are invariant under changes in the true behavior of these shocks. Without this extreme assumption, the kinds of policy simulations called for by the theory of economic policy are meaningless" (Lucas, 1976, p. 25). He adds that "Everything we know about dynamic economic theory indicates that this presumption is unjustified" (p. 25).

1. "Private sector behavior is influenced in many ways by expectations of future variables. If changes in government behavior change these expectations, models that ignore such links from government behavior via private expectations to private behavior are likely to forecast poorly and lead to misleading conclusions being drawn from policy simulations. This conclusion does not require Muth-rational expectations *per se,* only some direct effect of government behavior on private expectations. The assumption of Muth-rational expectations provides the additional hypothesis that the link between private sector expectations and government behavior comes through the private sector's knowledge of the true structure of the model, including the parameters that describe government behavior" (Buiter 1980, pp. 35-36).

In evaluating the Lucas critique, Gordon (1976) discusses that Lucas' condemnation of economic policy evaluation is not only pessimistic but is rather considerably overstated. He concludes, (p. 47), that "the effects of *some* policy changes can be determined if parameter shifts are allowed and are either (a) estimated from the response of parameters to policy changes within the sample period, or (b) are deduced from *a priori* theoretical consideration". It is now agreed the Lucas critique has made it quite clear that policy evaluations cannot satisfactorily be performed within the *classical* theory of economic policies. In other words, existing econometric models cannot be used for examination of policy changes since the structural parameters, such as the propensity to consume out of wealth or the interest elasticity of money demand, would likely change as policy changes.[1]

Nonetheless, the Lucas critique has inspired new empirical research work in macroeconomics to identify the *deep structural parameters.* It is agreed (see, for example, Sargent 1982) that once the truly structural parameters in an econometric model (i.e. tastes and technology in utility and production functions) are identified, the response of consumers and producers to policy actions can be deduced. Instead of estimating the structural relations, the parameters of, for example, utility function are estimated in this new approach. Intertemporal optimization lies at the heart of this approach and the parameters (of, say, utility function) are estimated from the first order conditions which are in fact the Euler equation.[2]

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1. For other critical comments on the Lucas critique, see Sims (1982, 1987) which specifies the very restrictive conditions under which the identification of *differences* between system behavior under different policy rules with *changes* in system behavior when the policy rule is changed at some point in future is valid. Tony Lawson (1995) offers a valuable contribution to the Lucas critique. He generalizes the Lucas critique beyond the simple implications for policy simulations using already derived econometric models. He attempts to extend the critique to the "endeavours of constructing and estimating such models in the first place, if using observations recorded in periods in which policy rules have been frequently changing" (p. 258).
2. 1n this regard, Hall and Mishkin (1982) have estimated the rational expectations life-cycle consumption model using panel data in order to arrive at the parameters' estimation of a utility function. This method can also be used to test restrictions. For example, by testing restrictions imposed by the underlying model of intertemporal optimization, Hall and Mishkin (1982) have found that about 20 percent of consumers in their sample do not satisfy the first order Euler conditions, implying that they may be regarded as being liquidity constrained. For a further discussion of this point and a strong critical analysis of the Euler equation approach (mainly with reference to its identification problems), see Garber and King (1983).

Despite the fact that the rational expectations hypothesis and particularly the contributions by Lucas, have had profound effect on policy modeling and econometric practice as well as on the mechanism of building policy-oriented macro-econometric models, it is by no means acceptable that all areas of macro-econometric behavior should be modeled according to forward-looking or rational expectations hypothesis. As Currie and Levine (1993a, pp. 1-2) have pointed out “Even if we were convinced that all economic agents behaved in this way, backward-looking relationships can be regarded as empirically acceptable approximations if the influence of the past on current decisions greatly outweighs the influence of future (rational) expectations. The stability of many macroeconomic relationships in the face of many changes in regime indicates that, for whatever reason, the Lucas critique may not be all pervasive”.

3. Time-inconsistency and the Optimal Control of Macro-econometric Models with Rational Expectations

The problem of ensuring consistency in dynamic choice was first addressed in a seminal paper by Strotz (1956). The problem for Strotz was not to explore the implications of rational expectations for optimal policy-making. He was mainly concerned with examining the problem of optimal choice among a number of alternative time-paths for consumption when consumer's taste changes. For Strotz the crucial question was that: “if [a consumer] is free to reconsider his plan at later date, will he abide by it or disobey it- *even though his original expectations of future desires and means of consumption are verified?*” (p. 165, emphasis in the original). His answer is that the optimal plan of future behavior chosen as of a given time “may be inconsistent with the optimizing future behavior of the individual *(the interlemporal tussle)*. In this case, (1) the conflict may not be recognized and the individual will then be *spend-thrifty* (or *miserly),* and his behavior being inconsistent with his plans, or (2) the conflict may be recognized and solved either by (a) a *strategy of pre-commitment,* or (b) a *strategy of consistent planning”* (p. 180, emphasis in the original). It is interesting to note that the concept of *pre-commitment,* which is now widely used in optimal control of macro-econometric models with rational expectations, was originated and carefully applied by Strotz in 1956.

Note that for the case of consumer behavior considered by Strotz, a contractual savings scheme could enable an individual to enter into pre-commitment to carry out the plan. It should also be noted that by the strategy of consistent planning, Strotz implies that the optimal current decisions are based on the assumption that plans will be revised in the future. The dynamic programming approach can essentially be used to formulate a consistent planning by imposing the principle of optimality at each stage of planning.

The problem of inconsistency in dynamic choice was extended by Pollak (1968), Blackorby *et al* (1973), Pelag and Yaari (1973), and Hammond (1976), among others. However, it was the seminal work of Kydland and Prescott (1977) which first recognized the possibility of optimal policies in models with rational (forward-looking) expectations to become sub-optimal through the passage of time (see, also Prescott, 1977). This property, which is known as *time-inconsistency* or *dynamic inconsistency* in forward-looking models, has significant implications for optimal control applications to economic policy-making practice.

Kydland and Prescott (1977) have shown that when government economic policies affect the way in which the private sector's expectations are formed, an optimal future policy obtained in one period will not be optimal from the vantage point of that future period, implying that *ex ante* optimal policies become sub-optimal *ex post.* In other words, when the current state is a function of the announced future economic policies, time-consistent values for current and future policy-instruments are not optimal because a lower value for the objective (cost) function can be achieved by implementing a different value for the future policy-instruments. It is, therefore, to the advantage of the policy-maker to change the policies in subsequent periods which are optimal from the vantage point of a previous period. This conclusion is based on the assumption that economic agents take into consideration any relevant information (including the information on the intentions of policy-makers) in the process of optimizing their dynamic choice.

For *causal* or backward-looking models in which the current state is a function of both the past state variables and the current values of control variables, the application of the standard optimal control techniques, such as Bellman's dynamic programming, does not face any problem. However, since the standard dynamic programming does not accommodate the impact of future policies on current values of state variables, the principle of optimality breaks down for optimal control of *non-causal* or forward-looking models in which current state variables depend on the anticipated future states.

To explain the failure of the principle of optimality recall that in engineering optimal control the current state of a system in a non-stochastic environment is a function of the initial state and the sequence of policies applied, while the future state of the system depends upon the current policy and the current state. The private sector will respond to its own expectations of the future state of the economy resulting from the policy actions announced by the government. The principle of optimality maintains that "an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" (see Bellman, 1957). The assumption of forward-looking rational expectations hypothesis clearly violates the additive separability assumption of objective function inherent in the principle of optimality. When policy optimization unfolds, previous optimal policy recommendations may not appear to be optimal due to the observed variations in state variables resulting from the realization of forward-looking expectations. The dynamic programming solution, using backward recursive functional, does not provide the same answer as the global optimum values unless the model is *causal*. In other words, the time-consistent solution is not necessarily the same as the optimal solution unless the future decisions do not affect the current states, which could make the system causal.[1]

The above argument underlies the strong reservation of Kydland and Prescott (1977) on the application of optimal control theory to economic stabilization policies. They argue that "even if there is an agreed-upon, fixed social objective function and policy-makers know the timing and magnitude of the effects of their actions, discretionary policies, namely, the selection of that decision which is best, given the current situation and a correct evaluation of the end-of-period

1. For further discussions on this point, see Holly and Zarrop (1983), Levine and Holly (1987) and Holly and Hallett (1989). For an examination of the conflict between optimality and time-inconsistency, see Calvo (1978) who carefully examines the difference between the constraints facing the policy-maker in subsequent periods and what they were in the previous period.

position, does not result in the social objective function being maximized. The reason for this apparent paradox is that economic planning is not a game against nature, but, rather, a game against rational economic agents. We conclude that there is *no* way control theory can be made applicable to economic planning when expectations are rational" (p. 473).

If Kydland and Prescott's recommendation is not to attempt to select policies optimally, how should then policies are selected? They join Lucas (1976) in saying that "economic theory [should] be used to evaluate alternative policy rules and that one with good operating characteristics be selected. It is probably preferable that rules be simple and easily understood, so it is obvious when a policy-maker deviates from optimum policies. There could be institutional arrangements which make it a difficult and time-consuming process to change the policy rules in all but emergency situations" (p. 487).

We now examine the pattern of research work which has developed in response to the new outlook towards optimal policy design originated by the seminal work of Kydland and Prescott. The underlying point is that the optimal policy choice is formulated as a dynamic game between intelligent players, i.e. policy-makers and the private sector. Under such circumstances, the behavior of private sector is conditioned by its perception of the nature of the control to be applied. When the policy-maker announces the policies, and when private sector's expectations are formed, there is always an incentive for the policy-maker to renege on the previously announced policies (time-inconsistent optimal policies); and since this can be anticipated by the private sector, the announced policies lack credibility in the absence of a pre-commitment mechanism. A simple solution follows that the government's discretionary power should be taken away by binding it to a fixed policy rule.

It is important to note that the failure of the standard optimal control theory in obtaining optimal policy sequence for dynamic economic systems with forward-looking expectations is not the result of the structural shortcoming of control theory in handling systems which actively react to control signals. One can argue that the prime interest of many econometricians and mathematical economists in the late 1960's and early 1970's were basically in the automatization of optimal economic planning, the computations of optimal policies in econometric models, and the examination of mathematical properties of optimal policy and state trajectories. This overshadowed the very basic argument of the structural differences between physical and economic systems and hence largely ignored those properties specific to economic systems which usually constrained the applications of engineering control theory. Otherwise, mathematical economists and econometricians did not have to wait until the 1980's and early 1990's to approach the dynamic optimization of economic systems from a game theoretic point of view. Mathematical control theory had so much to offer in game-theoretic control theory as early as the late 1950's.[1]

In the context of a two-person (government and the private sector), non-cooperative closed-loop game in which policy rules at any time are *functions* of the state at that time and thus of the new information set which becomes available, each player's policies affect the state and, therefore, influence the future policies via the closed loop or feedback mechanism. Each player minimizes his cost function by choosing his optimal policies subject to equations of motion and the rules of the game (for example, *Staekelberg* or *Nash* assumptions for single-stage or one-shot games).

With regard to the hierarchical structure of the Stackelberg games in which a leader (the government) can impose his actions on a follower (the private sector), the dominant player anticipates the reactions of the follower to the announced policies and then optimizes accordingly. The government optimizes subject to the private sector's first-order conditions for optimality. However, Miller and Salmon (1983, 1984) have shown that, in contrast to the single-controller, the optimal rule cannot be expressed as a linear time-invariant feedback even if an open loop Stackelberg game is assumed. They have demonstrated that, in this case, optimal policies are either a liner *time-varying* contingent rule or a type of *integral control*.

In this regard, Levine and Currie (1984) argue that such rules are attractive because they avoid feedback on other possible unknown variables. The open loop Stackelberg equilibrium exists if the leader is committed to pursue his announced optimal policies. As discussed earlier, under the assumption of unrestricted discretion, the dominant player can benefit by reneging on the announced initial plan since the leader knows that his announced future plans can significantly

1. See Berkowitz and Fleming (1957), Ho, Bryson and Baron (1965) and Behn and Ho (1968) for an approach based on differential games and optimal pursuit-evasion strategies.

influence the follower. However, after the expectations are formed, bygones are bygones and the departure from the *ex ante* optimal policies becomes more profitable. Note that the plan which involves the combination of announcement and reneging is dynamically consistent since there is no incentive to depart from it (the *perfect cheating* solution in Currie and Levine, 1985).

The formal procedure of incorporating rational expectations into the optimal control problem is to partition the state vector into predetermined state variables and a vector of non-predetermined forward-looking (free or jump) variables.[1] Levine and Currie (1983, 1984) and Currie and Levine (1983, 1984a,b,c) have successfully attempted to derive a solution to control rules in models of rational expectations. They have found that a feedback rule on the predetermined state variables must be in the form of either a linear time-varying rule on the current value of the predetermined state variables or a time-invariant rule on the current and past values of such variables.

When the two players (government and the private sector) interact according to Nash assumption, each player takes the other's actions as given and thus each has equal status in the game. In other words, under the Nash assumption, each player has no influence on the behavior of others and hence an equilibrium position is one that provides no incentive for players to move. Whilst the Stackelberg assumption appears to be more satisfactory in explaining government-private sector's behavior in macro-econometric models, the policy-maker, under the Nash assumption, should ensure that the expectations are consistent with the optimized policies since with rational expectations hypothesis economic agents understand and correctly anticipate the policy-makers' policies. In an open-loop Nash game, each player minimizes his cost function subject to the system dynamics while treating as parametric his rivals' policy vector.

Currie and Levine (1985) provide a solution to the open loop Nash game in the context of optimal control theory. The Nash assumption ensures the identity of *ex ante* and *ex post* optimal policies since there will not be an incentive to renege on the announced polices. The Nash

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1. For the exact definition of these terms, see Currie and Levine (1982) and Buiter (1984).

game precludes time-inconsistency by making players ignore the effects of their decisions on the rivals' policy rules.[1]

4.Time-inconsistency, Reputation and the Stochastic Environment

Repeated games involve memory and thus current strategies depend upon the past history of the game. The resulting notions of *reputation* and *credibility* may prevent the government to depart from the announced optimal plan. The problem of reputation building and its implications for time-inconsistency are reported in Barro and Gordon (1983). Backus and Driffill (1985a,b), Barro (1976). Currie and Levine (1993a,b) provide a number of important extensions on this issue.

The essential point is that designing a superior policy to the Nash strategy is possible only when the policy-maker is concerned with reputation. The contributions on reputational considerations can be regarded as an important development in the time-inconsistency literature. The mechanism of reputation building is an interesting complex problem. For example, Backus and Driffill (1985a,b) refer to the implausible length of *time* that the government and the private sector have been playing the game in the UK experience of anti-inflationary policies. The UK government's commitment to such policies has been met with private sector skepticism to expect higher inflation. The behavior of the private sector has not reflected the commitment of government to anti-inflationary policies.

Much of what has been said earlier for the deterministic case can be applied to the more realistic stochastic environments. Levine and Currie (1984) have shown that the *certainty equivalence* property of linear systems with quadratic cost function applies equally to the optimal control problems with rational expectations. In other words, the optimal control rule is the same for the stochastic and deterministic cases when the system is linear and the cost function is quadratic. In a further contribution to optimal control of stochastic rational expectations models, Currie and Levine (1985) have pointed out the significance of the discount parameter on the time-inconsistency property of the optimal policy in a stochastic environment. When the policy-maker gives higher weight to the future events the incentive to renege on the announced policies declines. The

1. For the first comprehensive survey of Nash and Stackelberg equilibrium strategies in dynamic games, see Cruz (1975).

policy-maker should weigh the advantages of reneging against the costs of pursuing the inferior time-consistent rule with respect to future shocks. As Currie and Levine (1985) conclude, if the rate of discount is not too high, the future cost will outweigh the gains from reneging. This implies that the policy-maker has an incentive to adhere to the ideal rule and thus avoid the temptation to cheat. Since the private sector is aware of this, the ideal rule is credible and sustainable.

It is well-known that stochastic shocks transform a deterministic policy game into a repeated game. The policy-maker is, therefore, more inclined to invest in its reputation to secure long-term policy gains rather than short-term gains from reneging. However, as Currie and Levine (1985) have pointed out “the key assumptions are that governments last forever and that the private sector never forgets past inconsistencies so reputations cannot be re-established”.

There are arguments suggesting that in a stochastic environment an observed policy change should be decomposed into two components: one arising from the optimal response to random shocks (stabilization component) and the other, arising from the time-inconsistency of the optimal policies (strategic component). Although such an uncertainty about the cause of an observed policy change is beneficial to the policy-maker by exploiting the private sector's uncertainties, the private sector might become more inclined to distrust any government announcements due to the resulting higher levels of confusion. Canzoneri (1985) provides an example of this type of problems by examining monetary policy games and the role of private information.

A stochastic environment sheds more light on the question of simple rules versus full optimal feedback rules. Recall the earlier discussion on Kydland and Prescott (1977) who strongly advocated simple rules on the basis that they have good operational characteristics and can easily be understood, allowing private sector to monitor policy-makers' deviations from the announced policies. The second property is of crucial importance particularly when the private sector's behavior depends strongly on their *understanding* of government's announced policies.

As Levine and Currie (1984) have shown, simple rules which are specified at the initial period to be linear time-invariant feedbacks on the state variables do not satisfy the property of certainty equivalence. One might, therefore, argue that the performance of these rules becomes a function of the nature of the future stochastic unknown disturbances. This can significantly reduce the desirability of simple rules in practical implementations. In a series of papers, Currie and Levine (1983, 1984a,b,c) and Levine and Currie (1983, 1984) have reported the possibility of identifying simple rules that perform well in the presence of different random disturbances. Since simple rules are basically designed to protect against random exogenous shocks, they are sensitive to the nature of such disturbances. The information concerning a group of shocks specific to an economy is, therefore, of vital importance in designing simple rules.

5. Rational Expectations and Optimization in Econometric Modeling in Practice

Lessons from the repeated-game literature together with the advances in reputational equilibrium, strategic behavior involving memory, stochastic environments in game theory and information structure of games have substantially enriched the literature on economic policy optimization with forward-looking expectations. An important question is how these ideas are relevant to as well as being useful in the practice of economic policy-making in the real world

In contrast to major *theoretical* advances in macroeconomics during the second half of the 20th Century, it appears that macroeconomists in business and government largely continued to base their forecasting and policy analysis on conventional medium to large-scale macro-econometric models. To explain the disparity between the theoretical macroeconomics and applied macro-econometrics one may argue that the nature and complexities of these theoretical developments have been of the sort that cannot be quickly adopted by applied macro-econometricians.

Mankiw (1988, p. 437-438) provides an analogy from the history of science to support the above argument: “The Copernican system held out the greatest promise for understanding the movements of the planets in the simplest and intellectually most satisfying way. Yet if you had been an applied astronomer, you would have continued to use the Ptolemaic system. It would have been fool-hardy to navigate your ship by the more promising yet less accurate Copernican system. Given the state of knowledge immediately after Copernicus, a complete separation between academic and applied astronomers was reasonable and indeed optimal”. It appears that Mankiw's argument is not very convincing since it is based on the assumption that the real value of theoretical advances in macroeconomics will ultimately be judged by whether they prove to be useful to applied econometricians. The validity of this argument can be seen only through the passage of time. In other words, what Mankiw is really suggesting is that we should wait to see its "success".

Computational difficulties associated with optimal control solutions for models with forward expectations have been one of the real obstacles in applied work. Recall that an optimization algorithm usually requires a Jacobian matrix of first derivatives of targets with respect to control variables in all periods. In contrast to conventional backward-looking models, where target variables respond only to current and lagged values of control variables, in models with forward expectations, such derivatives require the evaluation of model-consistent forward expectations.[1]

Optimization of large scale non-linear models with rational expectations can become excessively expensive from computational point of view. Obtaining linear representations of the original non-linear models is a reasonable attempt to deal with non-linear models. For example, Christodoulakis, Gaines and Levine (1991) have linearized the London Business School model before developing a methodology to design optimal fiscal and monetary policies for large econometric models with rational expectations.[2]

Despite its immediate low practical returns, the optimal control of macro-economic models with forward-looking expectations has considerable potential in advancing the econometric practice of model building. By emphasizing micro-foundations as well as macro-economic policy coordination (at national and international levels),

1. For a further discussion of this point, see Wallis (1995).

2. For earlier work on implementations of rational expectations in the London Business School model, see Budd *et al* (1984). See also Levine (1988), Levine and Currie (1987) and Christodoulakis and Levine (1988) for further elaboration on this point. Hall (1986) presents a Nash-type computation in a model with a single forward expectations variable. For a Stackelberg approach in deriving an inflation-unemployment trade-off by optimal control in a model with three forward expectations variables, see Wallis (1980, chapter 3) and for an appraisal of such different computational methods, see Fisher (1992, chapter 7). Both the Bank of England [Easton and Matthews (1984)] and the Treasury [Spencer (1984)] have reported experimenting with models incorporating rational expectations. Similarly, Darby (1984) reports on the National Institute's work on macro-economic models with rational expectations. And finally, for an optimal control software for rational expectations models, see Gaines, al-Nowaihi and Levine (1989).

macro-econometric modeling with rational expectations provides a better understanding of the real economy at work. It may also provide a profound impact on changing the outlook and the way economic policies are being designed. The two concepts of time-inconsistency and credibility are the most important outcomes when optimal control of an economic system is viewed as a dynamic game between intelligent players.

6. Summary and Concluding Remarks

The main concluding remarks are stated below.

1. Attempts to accommodate rational expectations in control theory applications to economic policy optimization necessarily involve game theoretic ideas. The amount of information that each player (government as the controller and the private sector) has on the constraints of the other player is an important factor in deriving the optimal solution. The fact that the public has considerable information on government's objectives and constraints in democratic environment significantly limits the success of any set of optimization policies. Moreover, despite being Pareto-optimal, the assumption of a cooperative game is not always realistic. On the other hand, there is no possibility of stopping a player from reneging on the cooperative solution or from cheating in a non-cooperative setting. Although the loss of reputation when the government reneges on the announced policies might ease the severity of this problem, the non-optimality of the policies adopted will remain a crucial issue.

2. When the government plays the dominant role in a single-stage non-cooperative game, the optimal policies will become time-inconsistent and thus violate the principle of optimality. The incentive to renege on the announced policies transforms the conventional optimal control of economic systems to the problem of finding policies which are optimal within the subset of credible and time-consistent policies. Lessons from the repeated game literature together with the advances in reputational equilibrium, strategic behavior involving memory, stochastic environments in game theory, and information structure of games have substantially enriched this literature.

3. Despite its immediate low practical returns, the optimal control of macro-econometric models with forward-looking expectations have considerable potentials in advancing the econometric practice of

model building. By emphasizing micro-foundations as well as macroeconomic policy coordination (at national and international levels), macro-econometric model-building with rational expectations provides a better understanding of the real economy at work. It may also provide a profound impact on changing the outlook and the way economic policies are being designed. The two concepts of time-inconsistency and credibility are the most important outcomes when optimal control of an economic system is viewed as a dynamic game between intelligent players. These concepts will continue to play a significant role in an econometric approach towards optimal economic planning.

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